Math 262 **Test #2** **Spring 2005**

1. (6 pts) Write the summation

$$\frac{1}{2} - \frac{2}{3} + \frac{4}{4} - \frac{8}{5} + \frac{16}{6}$$

using summation notation.

2. (10 pts) Use the well known formula for $\sum_{k=1}^{n} k$ to write a closed-form expression for

$$\sum_{k=1}^{n}(2k+3) = 5 + 7 + 9 + 11 + \cdots + (2n+3).$$

3. (12 pts) Use mathematical induction to prove that

$$\sum_{k=1}^{n} k(k+1) = \frac{n(n+1)(n+2)}{3}$$

for all integers $n \geq 1$.

4. (12 pts) Define a sequence of numbers $\{a_n\}$ as

$$\begin{aligned} a_1 &= 1 \\ a_n &= 2a_{n-1} + 5, \text{ for all } n \geq 2 \end{aligned}$$

Use mathematical induction to prove that $a_n = 3 \cdot 2^n - 5$ for all $n \geq 1$.

5. (10 pts) Let $A = \{1, 2, 4, 8\}$, $B = \{1, 3, 5, 7\}$, and $C = \{2, 3, 5, 7\}$ and let the universal set be $\{1, 2, 3, 4, 5, 6, 7, 8\}$. Find each of the following sets.

   (a) $A \cup B$

   (b) $B \cap C$

   (c) $(A \cup B) - (A \cap B)$

   (d) $(B \cap C)^c$

6. (10 pts) Suppose we have a program `ZERO` that can read any program P and any data X (both expressed in binary) and determine whether P will print 0 when run with input X. (Program `ZERO` will print either "yes" or "no.") Explain why this will lead to a contradiction, and therefore why no such program `ZERO` can exist. Use an argument parallel to the argument used in the Halting Problem. Be sure that your argument is clear and precise. You may add a drawing if that will help, but be aware that the drawing will not be self-explanatory.

7. (10 pts) Suppose that two red cards and two black cards are selected from a deck of cards. Label the cards R_1, R_2, B_1, and B_2. They are placed face down and then two of them are selected at random and turned over.

   (a) List all possible choices of the two cards that are turned over.

   (b) What is the probability that the two cards that are turned over are the same color?

8. (10 pts)

   (a) How many integers are there from 1 to 10000 that are *not* multiples of any of the primes 29, 37, or 89?

   (b) If an integer from 1 to 10000 is selected at random, what is the probability that it *is* a multiple of 29, 37, or 89?

9. (10 pts) A bag contains 5 red marbles and 5 green marbles. A person chooses three marbles at random from the bag. He wins a prize if the three that he chooses are all the same color. What is the probability that he wins a prize?

10. (10 pts) Use the Binomial Theorem to expand $(a + 2b)^5$.